Parameters and Equations of the Lake Model SALMO

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Contents

1 The lake model SALMO

The "Ecological Lake Model SALMO" (Simulation by means of an Analytical MOdel) is a dynamic model, originally developed at TU Dresden, Institute of Hydrobiology. It describes essential parts of the aquatic foodweb of lakes and reservoirs.

The system of ordinary differential equations originates from the habilitation thesis of Benndorf (1979) who modelled annual time-dependend development of phytoplankton (two groups), zooplankton, oxygen, nutrients (N and P) and external detritus of the water body, based on field observations and laboratory experiments. First implementations in Fortran and HPL (Hewlett Packard Language) have been developed by Recknagel (Recknagel, 1980; Recknagel and Benndorf, 1982) and were used for numerous theoretical and practical studies (e.g., Benndorf and Recknagel, 1979b,a, 1982; Benndorf *et al.*, 1985; Petzoldt and Recknagel, 1991). Since then, several versions, implementations and spin-offs followed.

This R package aims to make an "almost original version" of the model publicly available under the GPL 2 and to foster further development. Its source code is derived from an independent implementation of the system of equations of model version SALMO II (Benndorf, 1988). The JAVA version of Dietze and Planke (Willmitzer *et al.*, 1998) was the followed by the C version SALMO-1D of Rolinski (Petzoldt and Siemens, 2002; Rolinski *et al.*, 2005; Petzoldt *et al.*, 2005; Petzoldt and Uhlmann, 2006), that allowed coupling to hydrophysical models such as LAKE (Baumert *et al.*, 1989, 2005b,a) or GOTM (Umlauf *et al.*, 2007). During this time, the system equations underwent fundamental generalisation (same equation for all seasons) and slow but steady evolution.

Recently, Sachse *et al.* (2014) coupled SALMO-1D to a macrophyte module based on the model PCLake (Janse and van Liere, 1995; Janse *et al.*, 1998) to simulate growth of and interaction effects with submersed water plants.

The code is now maintained by Thomas Petzoldt at TU Dresden, Institute of Hydrobiology. More information can be found on:

- `http://www.simecol.de/salmo/`
- `http://rlimnolab.r-forge.r-project.org/`
- `http://tu-dresden.de/Members/thomas.petzoldt/`

Note that all this is still work in progress, so please contact me before trying to run applications or investing your valuable work.

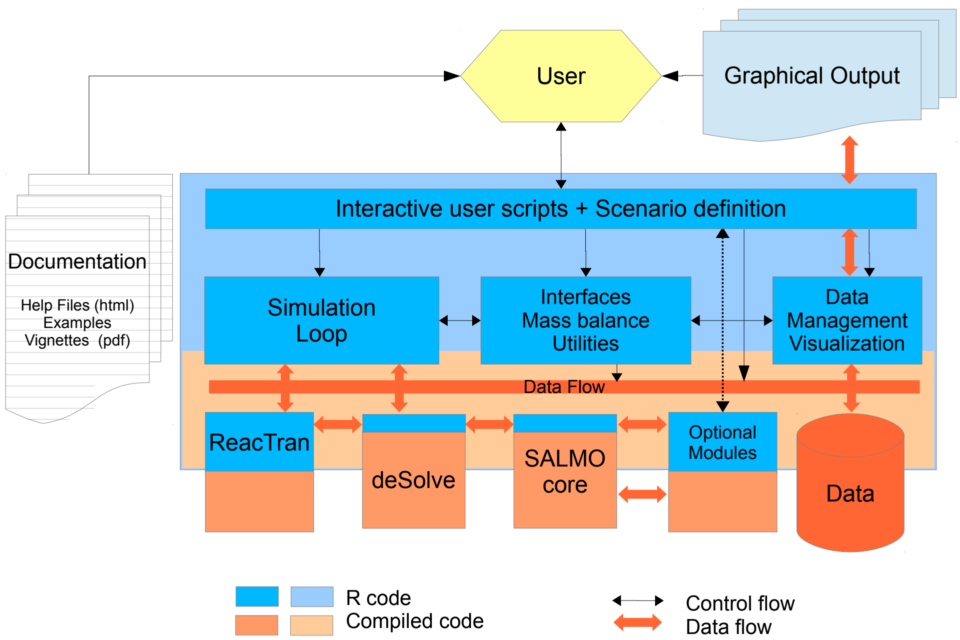

Figure 1: Concept of the R package rSALMO

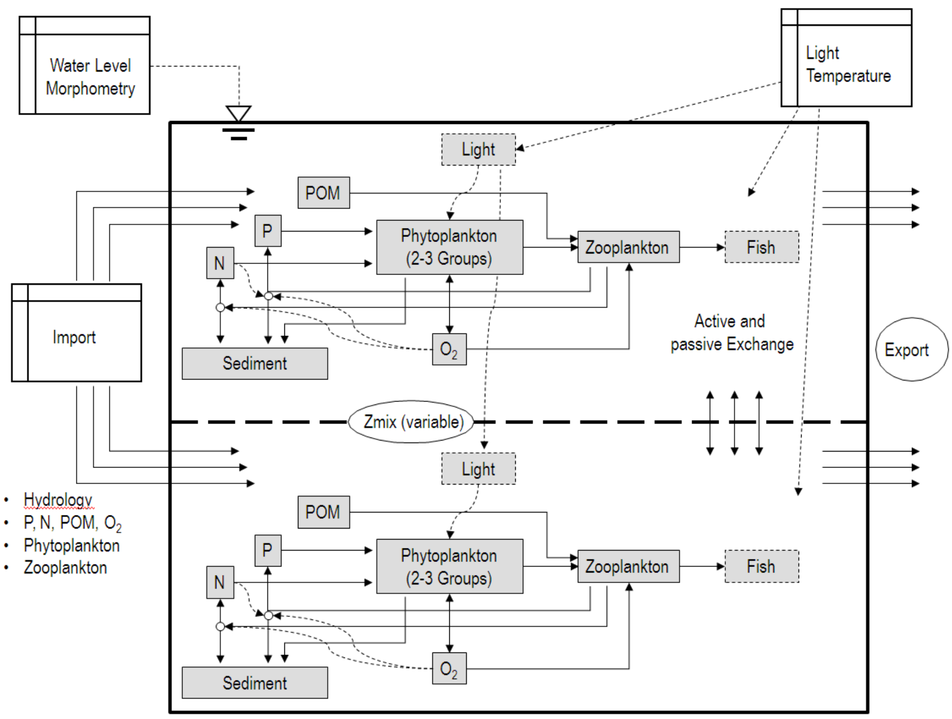

Figure 2: Simplified schematic representation of the two-layer configuration of lake model SALMO-2

2 Model parameters and internal variables

2.1 Constant model parameters

$ANSFMIN = 0.01$	minimal value of $ansf$ (release of inorganic nitrogen from sediment) at 0 °C ($g\ N\ m^{-2}\ d^{-1}$)
$APSFMAX = 7$	maximal value of $apsf$ (phosphorus release from sediment), if oxygen concentration $o < LINDEN$ ($mg\ P\ m^{-2}\ d^{-1}$)
$APSFMIN = 1$	minimal value of $apsf$ (phosphorus release from sediment), at saturation concentration of oxygen ($mg\ P\ m^{-2}\ d^{-1}$)
$AZMAX = 0.8$	maximal assimilation coefficient of zooplankton at very low ingestion rate (-)
$AZMIN = 0.4$	minimal assimilation coefficient at ($g = GMAX$) (-)
$DTA = 3.9$	parameters of the empirical relationship between egg development time of crustaceans and temperature (-)
$DTB = 0.15$	cf. DTA
$DTC = 0.26$	cf. DTA
$DTMIN = 5$	minimum value of egg development time of the crustaceans below of which the egg development time is neglected in the calculation of zooplankton growth (d)
$EPSD = 0.023$	specific light extinction coefficient of detritus ($m^2\ g^{-1}$)
$GI = 0.8$	inhibition factor of the ingestion rate due to light (-)
$GMAX = 1.3$	maximum value of g (specific ingestion rate of zooplankton) ($g\ g^{-1}\ d^{-1}$)
$GMIN = 0.26$	minimum value of g (specific ingestion rate of zooplankton) near 0 °C ($g\ g^{-1}\ d^{-1}$)
$KANSF = 0.004$	slope of the function $ansf(temp)$ (release of organic nitrogen from sediment) ($g\ m^{-2}\ d^{-1}\ °C^{-1}$)
$KAPSF = 1.25$	half saturation constant of the inverse function $apsf(o)$ (phosphorus release from sediment) ($g\ O_2\ m^{-3}$)
$KDEN = 0.045$	parameter of the dependence of denitrification on the supply of organic matter ($g\ cm^{-3}$)
$KMINER = 0.04$	mineralisation constant related to oxygen consumption by sinking phytoplankton (d^{-1})
$KMO = 0.35$	half saturation constant of the dependence of zooplankton mortality on zooplankton biomass ($g\ m^{-3}$)
$KPSED = 0$	specific scenario parameter for hard water lakes, sedimentation of phosphate by co-precipitation with calcite as daily percentage of the in-lake phosphate (-)
$KSEZA = 2.5$	oxygen concentration o, which causes 50% of the maximal oxygen consumption by the sediment ($g\ O_2\ m^{-3}$)

$KSRFMAX = 4.3$	critical value of $ksrf$ (corrected strong rain factor), above the underwater light climate is reduced due to erosion-induced turbidity (-)
$KXG = 5$	half saturation constant of the relationship between ingestion rate of zooplankton and food ($g\ m^{-3}$)
$KXMIN = 2.5$	theoretical minimum of kx and kxn (half saturation constant of the inverse relationship between photosynthesis rate and biomass of phytoplankton at nitrogen limitation of phytoplankton growth) ($g\ m^{-3}$)
$KZMIN = 4$	theoretical minimum value of kz (half saturation value of the inverse relationship between ingestion rate and biomass of zooplankton) ($g\ m^{-3}$)
$LGH = 0.4$	parameter of the dependence of the half saturation value kz_i on phytoplankton biomass at high biomass of x_i (-)
$LGL = 5.76$	parameter of the dependence of the half saturation value kz_i on phytoplankton biomass at low biomass of x_i (-)
$LINDEN = 1$	oxygen threshold below which denitrification occurs ($g\ O_2\ m^{-3}$)
$LXH = 0.1$	parameter of the dependence of the half saturation value kx on the phosphate concentration at high phosphate concentration (-)
$LXHN = 209.56$	parameter of the dependence of the half saturation value kxn on the concentration of inorganic nitrogen at high inorganic nitrogen concentration (-)
$LXL = 2.78$	parameter of the dependence of the half saturation value kx on on the phosphate concentration at low phosphate concentration(-)
$LXLN = 19.04$	parameter of the dependence of the half saturation value kxn on the concentration of inorganic nitrogen at low inorganic nitrogen concentration (-)
$MGH = 1.5$	parameter of the dependence of the half saturation value kz on phytoplankton biomass at high biomass of phytoplankton (-)
$MGL = 0.41$	parameter of the dependence of the half saturation value kz on phytoplankton biomass at low biomass of phytoplankton (-)
$MOMIN = 0.015$	rate of zooplankton mortality near 0 °C at zooplankton biomass z much higher than KMO (d^{-1})
$MOT = 0.006$	slope of the function $mortz(temp)$ (°$C\ d^{-1}$)
$MXH = 1.55$	parameter of the dependence of the half saturation value kx on the phosphate concentration at high phosphate concentration (-)

$MXL = 0.39$	parameter of the dependence of the half saturation value kx on the phosphate concentration at low phosphate concentration (-)
$OPTNP = 0.0072$	optimum N/P mass ratio (-)
$PF = 1$	preference factor for ingestion of detritus by zooplankton (-)
$R = 2$	parameter of the dependence of the ingestion rate of zooplankton on water temperature (-)
$RAT = 0.7$	ratio of soluble to total phosphorus in zooplankton faeces (-)
$RATF = 0.7$	ratio of soluble to total phosphorus in fish excrements and during remineralization of dead zooplankton (-)
$RATN = 0.7$	ratio of soluble to total nitrogen in zooplankton faeces (-)
$RATNF = 0.7$	ratio of soluble to total nitrogen in fish excrements and during remineralization of dead zooplankton (-)
$RL = 0.9$	factor for light reflection at the water surface (-)
$RLW = 0.2$	factor for light reflection at the water during winter stagnation (ice cover) (-)
$RXMF = 0.3$	fraction of the gross photosynthesis rate which is consumed by respiration additionally to the basis respiration (-)
$RZMIN = 0.08$	respiration rate of zooplankton at optimal temperature for feeding but without food supply (d^{-1})
$RZOPT = 0.22$	respiration rate of zooplankton at optimal temperature for feeding and maximum ingestion rate (d^{-1})
$RZTMIN = 0.05$	respiration rate of zooplankton near O °C and optimal food supply (d^{-1})
$SF = 1.0$	heuristic sediment fucussing parameter (-)
$SEZMAX = 0.4$	maximum oxygen consumption by the sediment ($g\ O_2\ m^{-3} d^{-1}$)
$EPSR = 0.2668$	specific light extinction coefficient due to turbidity and erosion at strong rain events ($m^2\ g^{-1}$)
$TOPTZ = 20$	optimal temperature for feeding activity of the zooplankton (°C)
$UXZD = 0.75$	factor of the physiological utilisation of detritus by zooplankton (-)
$VD = 0.2$	net sinking velocity of detritus ($m\ d^{-1}$)
$VMIG = 0.15$	net migration velocity of zooplankton from hypolimnion to epilimnion ($m\ d^{-1}$)
$WPKX = 12.5$	point of inflexion of the function $kx(p)$ ($mg\ m^{-3}$)
$WPKZ = 8.6$	point of inflexion of the function $kz(x)$ ($g\ m^{-3}$)
$YD = 2$	phosphorus related 'yield' coefficient of detritus (g wet weight/$mg\ P$)

$YND = 285$	nitrogen related 'yield' coefficient of detritus (g wet weight/g N)
$YNX = 57$	nitrogen related yield coefficient of phytoplankton (g wet weight /g N)
$YOX = 3.75$	oxygen equivalent of the biomass (g wet weight/g O_2)
$YZN = 110$	nitrogen related yield coefficient of zooplankton (g wet weight/g N)
$YZP = 0.8$	phosphorus related yield coefficient of phytoplankton (g wet weight/mg P)

2.2 Phytoplankton parameters

$EPSX = (0.0368, 0.046, 0.046)$	specific light extinction coefficient of phytoplankton group i (m^2 g^{-1})
$KI = (28, 29, 29)$	half saturation constant of the dependence of the photosynthesis rate of phytoplankton group i on light (J cm^{-2} d^{-1})
$KN = (0.0123, 0.0123, 0.0095)$	half saturation constant of the dependence of the photosynthesis rate of phytoplankton group i on nitrogen at minimal phytoplankton biomass (g N m^{-3})
$KP = (1.7, 1.7, 9.5)$	half saturation constant of the dependence of the photosynthesis rate of phytoplankton group i on phosphorus at minimal phytoplankton biomass (g N m^{-3})
$KPF = (1.1, 4, 0)$	half saturation value of the dependence of the preference factor of zooplankton pf_i for phytoplankton group i on availability of the other groups (g m^{-3})
$PFC = (0, 0.3, 1)$	preference factor for ingestion of phytoplankton by zooplankton (-)
$PFX = (0.1, 3, 0)$	threshold of the other phytoplankton groups above which pf_i is decreasing (g m^{-3})
$PHOTXMAX = (1.7, 1.8, 3.5)$	maximum value of $photx_i$ (gross photosynthesis rate of group i) at optimal conditions (d^{-1})
$PHOTXMIN = (0, 0.17, 0.35)$	values of $photx_i$ near to 0 °C at optimal light and nutrient conditions (d^{-1})
$RXTMIN = (0, 0.02, 0.02)$	rate of basis respiration of phytoplankton group i near 0 °C (d^{-1})
$RXTOPT = (0.057, 0.06, 0.06)$	rate of basis respiration of phytoplankton group i at the group specific optimum temperature (d^{-1})
$TOPTX = (25, 20, 25)$	optimum temperature for phytoplankton group i (°C)
$UXZ = (1, 1, 1)$	factor of the physiological utilisation of phytoplankton group i by zooplankton (-)
$VS = (0.05, 0.1, 0.1)$	net sinking velocity of phytoplankton group i (m d^{-1})
$YX = (1, 0.8, 0.41)$	phosphorus related yield coefficient of phytoplankton group i (g wet weight / mg P)

2.3 Internal variables of the model

$ansf$	release rate of anorganic nitrogen from sediment ($g\ N\ m^{-2}\ d^{-1}$)
$apsf$	release rate of anorganic phosphorus from sediment ($mg\ P\ m^{-2}\ d^{-1}$)
$assiz$	assimilation rate of zooplankton ($g\ g^{-1}\ d^{-1}$)
az	assimilation coefficient (g food assimilated / g food ingested)
bd	sedimentation rate of detritus (d^{-1})
bx_i	sedimentation rate of phytoplankton group i (d^{-1})
$dgraz$	loss of detritus by zooplankton grazing ($g\ m^{-3}\ d^{-1}$)
dt	egg development time of all crustaceans (d)
egg	reduction factor of the zooplankton growth rate due to low reproduction (-)
eps	extinction coefficient of light (photosynthetic active radiation between 400 and 700 nm) (m^{-1})
g	specific ingestion rate of zooplankton ($g\ g^{-1}\ d^{-1}$)
g_i	specific ingestion rate of phytoplankton group i by zooplankton ($g\ g^{-1}\ d^{-1}$)
$g3d$	specific ingestion rate of detritus by zooplankton ($g\ g^{-1}\ d^{-1}$)
gdb	term of the dependence of the ingestion rate in the dark on the biomass of the phyto- and zooplankton (-)
gdt	temperature term of the ingestion rate in the dark ($g\ g^{-1}\ d^{-1}$)
gl	specific ingestion rate of zooplankton g in the light ($g\ g^{-1}\ d^{-1}$)
hl	light hours per day (photo period) ($h\ d^{-1}$)
hwg_i	auxillary value for the calculation of the ingestion rate g_i ($g\ g^{-1}\ d^{-1}$)
$hwgd$	auxillary value for the calculation of gd ($g\ g^{-1}\ d^{-1}$)
$hwgd_i$	auxillary value for the calculation of gd ($g\ g^{-1}\ d^{-1}$)
$hwgdb_i$	auxillary value for the calculation of gdb (-)
$hwgdbd$	auxillary value for the calculation of gdb (-)
$hwgdd$	auxillary value for the calculation of gd ($g\ g^{-1}\ d^{-1}$)
$hwgl_i$	auxillary value for the calculation of gd and gl ($g\ g^{-1}\ d^{-1}$)
$hwgld$	auxillary value for the calculation of gd and gl ($g\ g^{-1}\ d^{-1}$)
$ired$	value of iin (incoming radiation) reduced by reflection ($J\ cm^{-2}\ d^{-1}$)
$iredz$	photosynthetic active light in depth $zvonj$ ($J\ cm^{-2}\ d^{-1}$)
$ksrf$	corrected factor for strong rain events or melting snow (correction with respect to reduced erosion in the drainage basin in dependence of the vegetation cover) (-)
kx	half saturation value of the inverse relationship between photosynthesis rate and phytoplankton biomass at phosphate limiting conditions ($g\ m^{-3}$)
kxn	half saturation value of the inverse relationship between photosynthesis rate and phytoplankton biomass at nitrogen limiting conditions ($g\ m^{-3}$)
kz	half saturation value of the inverse relationship between ingestion rate of zooplankton and zooplankton biomass ($g\ m^{-3}$)
kz_i	half saturation value of the inverse relationship between ingestion rate of zooplankton and phytoplankton biomass of group i ($g\ m^{-3}$)
kzd	half saturation value of the inverse relationship between ingestion rate of zooplankton and detritus ($g\ m^{-3}$)

lo	loading of the water body with oxygen-consuming organic matter ($g\ m^{-3}\ d^{-1}$)
$lsez$	loading of the water body due to to oxygen consumption by the sediment ($g\ m^{-3}\ d^{-1}$)
M	number of depth levels for the internal calculation of light and photosynthesis
$miner_i$	mineralisation coefficient related to oxygen consumption by sinking phytoplankton of group i (-)
$minerd$	mineralisation coefficient related to oxygen consumption by sinking detritus (-)
$mortz$	zooplankton mortality rate (d^{-1})
$nexkr$	rate of nitrogen remineralisation by living zooplankton ($g\ N\ m^{-3}\ d^{-1}$)
$nkons$	assimilation of inorganic nitrogen by phytoplankton ($g\ N\ m^{-3}\ d^{-1}$)
$nmort$	rate of nitrogen mineralisation from dead zooplankton and by fish ($g\ N\ m^{-3}\ d^{-1}$)
$nrem$	remineralisation of nitrogen ($g\ N\ m^{-3}\ d^{-1}$)
nsf	release of inorganic nitrogen from the sediment to the water body ($g\ N\ m^{-3}\ d^{-1}$)
o	state variable: oxygen concentration ($g\ m^{-3}$)
$okons$	oxygen consumption (without phytoplankton respiration) ($g\ m^{-3}\ d^{-1}$)
$oprod$	net oxygen production by living phytoplankton ($g\ m^{-3}\ d^{-1}$)
$pexkr$	rate of phosphate remineralisation by zooplankton excretion ($mg\ P\ g\ Z^{-1}\ d^{-1}$)
pf_i	preference factor for ingestion of phytoplankton group i by zooplankton (-)
$photx_i$	gross photosynthesis rate of phytoplankton group i (d^{-1})
$phoxl_i$	term of the relationship between photosynthesis rate of phytoplankton group i and light (-)
$phoxn_i$	term of the relationship between photosynthesis rate of phytoplankton group i and anorganic nitrogen (including dependence on phytoplankton biomass) (-)
$phoxns_i$	term of the relationship between photosynthesis rate of phytoplankton group i and the primary limiting nutrient(including dependence on phytoplankton biomass) (-)
$phoxp_i$	term of the relationship between photosynthesis rate of phytoplankton group i and phosphate (including dependence on phytoplankton biomass) (-)
$phoxt_i$	term of the relationship between photosynthesis rate of phytoplankton group i and temperature (d^{-1})
$pkons$	phosphate consumption by phytoplankton ($mg\ m^{-3}\ d^{-1}$)
$pmort$	rate of phosphate remineralisation from dead zooplankton or zooplankton eaten by fish (d^{-1})
$prem$	remineralisation of phosphate ($mg\ m^{-3}\ d^{-1}$)
$psed$	sedimentation of phosphate by coprecipitation with calcite or other minerals ($mg\ m^{-3}\ d^{-1}$)
psf	release of phosphorus from sediment to the water body ($mg\ m^{-3}\ d^{-1}$)
rx_i	respiration rate of phytoplankton group i (d^{-1})
rxt_i	base respiration rate of phytoplankton group i (d^{-1})
rz	respiration rate of zooplankton (d^{-1})
rzg	factor of zooplankton respiration rate

rzt	factor of zooplankton respiration rate
sat	oxygen concentration at 100% saturation ($g\ m^{-3}$)
$seza$	oxygen consumption of the sediment (per sediment surface area) ($g\ m^{-2}\ d^{-1}$)
$sumxpf$	Summe der Zustände x_i multipliziert mit pf_i
wx_i	growth rate of phytoplankton group i (d^{-1})
wz	growth rate of zooplankton (d^{-1})
$xgraz_i$	phytoplankton loss of group i due to zooplankton grazing ($g\ m^{-3}\ d^{-1}$)
xwa_i	growth of phytoplankton group i ($g\ m^{-3}\ d^{-1}$)
zmo	zooplankton mortality ($g\ m^{-3}\ d^{-1}$)
$zvonj$	depth of layer j (m)
zwa	zooplankton growth ($g\ m^{-3}\ d^{-1}$)

3 SALMO - System of Equations

3.1 Anorganic Nitrogen

Change =
- assimilation of organic nitrogen by phytoplankton
+ remineralisation of nitrogen
+ release of nitrogen from the sediment
+ ...

1.0 $$\frac{dn}{dt} = -nkons + nrem + nsf + nden \qquad \left[\frac{gN}{m^3} \cdot d\right]$$

1.2 $$nkons = \sum\left(\frac{wx_i}{YNX} \cdot x_i\right) + \frac{assiz}{YZN} \cdot RATN \cdot z\,; \quad i = 1, \ldots, nx$$

1.3 $$nrem = (nmort + nexkr) \cdot z$$

1.4 $$nmort = \frac{mortz \cdot RATNF}{YZN}$$

1.5 $$nexkr = \left(\left(\sum \frac{g_i}{YNX}\right) + \frac{g_d}{YND} + \frac{rz}{YZN}\right) \cdot RATN\,; \quad i = 1, \ldots, xn$$

1.7 $$nsf = \frac{ansfq - ansfs}{v} \cdot ased$$

1.8 $$ansfs = \begin{cases} NDSMAX \cdot \dfrac{n}{KNDS + n} \cdot KNDST^{(temp-4)}\,; & NDSSTART \leq t < NDSEND \\ ANSFMIN + KANSF \cdot temp\,; & \text{sonst} \end{cases}$$

$$ansfq = \begin{cases} ANSFMIN\,; & NDSSTART \leq t < NDSEND \\ 0\,; & \text{sonst} \end{cases}$$

3.8 $$nden = \frac{n \cdot KDEN \cdot lo}{KNDS + n}\,; \quad n > 0,\ 0 \leq LINDEN$$

3.2 Dissolved Inorganic Phosphorus

Change =
- phosphate uptake by phytoplankton
+ remineralisation
+ ...
+ release of phosphate from sediment

4.0 $$\frac{dp}{dt} = -pkons + prem + presp + psf \qquad \left[\frac{mg}{m^3} \cdot d\right]$$

4.3 $$pkons = \sum \left(\frac{photx_i}{YX_i} \cdot x_i \right) + \frac{assiz}{YZP} \cdot RAT \cdot z \; ; \quad i = 1, \ldots, nx$$

4.4 $$prem = (pmort + pexkr) \cdot z$$

$$presp = \sum \left(\frac{rx_i}{YX_i} \cdot x_i \right) \; ; \quad i = 1, \ldots, nx$$

4.5 $$pmort = \frac{mortz \cdot RATF}{YZP}$$

4.6 $$pexkr = \left(\left(\sum \frac{g_i}{YX_i} \right) + \frac{g_d}{YD} + \frac{rz}{YZP} \right) \cdot RAT \; ; \quad i = 1, \ldots, xn$$

4.9 $$psf = apsf \cdot \frac{ased}{v}$$

4.10 $b_1 = n - 0.3 \cdot LINDEN$

$b_2 = o + \frac{n}{0.3} - LINDEN$

$if(npsfmode = TRUE)$

$$apsf = \begin{cases} APSFMAX\ ; & b_1 \leq 0 \\ APSFMAX \cdot \dfrac{\frac{1}{b1}}{\frac{1}{(KAPSF - 0.3 \cdot LINDEN)} + \frac{1}{b_1}}\ ; & b_1 > 0 \end{cases}$$

$else$

$$apsf = \begin{cases} APSFMAX\ ; & b_2 \leq 0 \\ APSFMAX \cdot \dfrac{\frac{1}{b_2}}{\frac{1}{(KAPSF - LINDEN)} + \frac{1}{b_2}} + APSFMIN\ ; & b_2 > 0 \end{cases}$$

3.3 Light in the Water Column

7.0 $iredz(j) = ired \cdot \exp(-eps \cdot z(j))$ $\left[\frac{J}{cm^2} \cdot d\right]$

7.1 $ired = iin$

7.2 $eps = eps' + \left(\sum(EPSX_i \cdot x_i) + EPSD \cdot d\right)\ ;\quad i = 1, \ldots, nx$

7.3 $eps' = \begin{cases} EPSMIN + EPSR \cdot (ksrf - KSRFMAX)\ ; & qin > 0,\ ksrf > KSRFMAX \\ EPSMIN\ ; & \text{sonst} \end{cases}$

7.4 $ksrf = \left(1.5 + 0.5 \cdot \cos\left((t - 30) \cdot \frac{\pi}{180}\right)\right) \cdot srf$

7.5 $z(j) = z(j-1) \cdot \frac{zmix}{M}\ ;\quad M = \max\left(2, \frac{zmix}{ZLIGHT}\right)$

3.4 Phytoplankton

Change =
+ growth of phytoplankton
- sedimentation of phytoplankton
- grazing by zooplankton

9.0 $$\frac{dx_i}{dt} = xwa_i - xsed_i - xgraz_i \; ; \quad i = 1, \ldots, nx \qquad \left[\frac{g}{m^3} \cdot d\right]$$

9.1 $$xwa_i = wx_i \cdot x_i$$

9.2 $$wx_i = \frac{1}{M} \cdot \sum_{j=1}^{M} wx(j)_i$$

9.3 $$M = max\left(2, \frac{zmix}{ZLIGHT}\right)$$

9.4 $$wx(j)_i = photx_i(j) - rx_i(j)$$

9.5 $$photx_i(j) = phoxt_i \cdot phoxl_i(j) \cdot phoxns_i$$

9.6 $$phoxns_i = \begin{cases} phoxn_i \; ; & \frac{n}{p} \leq OPTNP \;\&\; NFIX_i < 1e-4 \\ phoxp_i \; ; & \text{sonst} \end{cases}$$

9.7 $$phoxt_i = \frac{(PHOTXMAX_i - PHOTXMIN_i)}{TOPTX_i} \cdot temp + PHOTXMIN_i$$

9.8 $$phoxl_i = \begin{cases} \frac{iredz(j) \cdot (1 - NFIX_i)}{KI_i + iredz(j)} \; ; & \frac{n}{p} < OPTNP \\ \frac{iredz}{KI_i + iredz} \; ; & \text{sonst} \end{cases}$$

9.9 $$phoxp_i = \frac{p}{x_i \cdot \left(\frac{KP_i}{kx} + \frac{p}{kx} + \frac{KP_i}{x_i} + \frac{p}{x_i} \right)}$$

9.10 $$kx = \begin{cases} LXL \cdot p^{MXL} \; ; & p \leq WPKX \\ KXMIN + LXH \cdot p^{MXH} \; ; & p > WPKX \end{cases}$$

9.11 $$phoxn_i = \frac{n}{x_i \cdot \left(\frac{KN_i}{kxn} + \frac{n}{kxn} + \frac{KN_i}{x_i} + \frac{n}{x_i} \right)}$$

9.12 $$kxn = \begin{cases} LXLN \cdot n^{MXL} \; ; & n \leq WPKX \cdot OPTNP \\ KXMIN + LXHN \cdot n^{MXH} \; ; & n > WPKX \cdot OPTNP \end{cases}$$

9.13 $$rx_i(j) = rxt_i + RXMF \cdot photx_i(j)$$

9.14 $$rxt_i = \frac{RXTOPT_i - RXTMIN_i}{TOPTX_i} \cdot temp + RXTMIN_i$$

9.16 $$xsed_i = bx_i \cdot x_i$$

9.17 $$bx_i = \begin{cases} \frac{VS_i}{zmix} \cdot SF \cdot (1 - aver) \; ; & tief > ZRES \\ 0 \; ; & \text{sonst} \end{cases}$$

9.18 $$xgraz_i = g_i \cdot z$$

9.19 $$g_i = \frac{hwg_i}{\sum hwg_i + hwg_d} \cdot g$$
$$g_d = \frac{hwg_d}{\sum hwg_i + hwg_d} \cdot g$$

9.20 $hwg_i = hwgd_i$
$$hwg_d = hwgd_d$$

9.21 $hwgd_i = gdt \cdot hwgdb_i$
$$hwgd_d = gdt \cdot hwgdb_d$$

9.22 $$gdt = (GMAX - GMIN) \cdot \left(\exp\left(-R \cdot \mathrm{abs}\left(\ln\left(\frac{temp}{TOPTZ}\right)\right)\right)\right) + GMIN$$

9.23 $$hwgdb_i = \frac{x_i \cdot pf_i}{z \cdot \left(\frac{KXG}{kz_i} + \frac{x_i}{kz_i} + \frac{KXG}{z} + \frac{x_i}{z}\right)}$$
$$hwgdb_d = \frac{d \cdot PF}{z \cdot \left(\frac{KXG}{kz_d} + \frac{d}{kz_d} + \frac{KXG}{z} + \frac{d}{z}\right)}$$

9.24 $$kz_i = \begin{cases} LGL \cdot (x_i \cdot pf_i)^{MGL}\ ; & x_i \cdot pf_i \le WPKZ \\ KZMIN + LGH \cdot (x_i \cdot pf_i)^{MGH}\ ; & x_i \cdot pf_i > WPKZ \end{cases}$$
$$kz_d = \begin{cases} LGL \cdot (d \cdot PF)^{MGL}\ ; & d \cdot PF \le WPKZ \\ KZMIN + LGH \cdot (d \cdot PF)^{MGH}\ ; & d \cdot PF > WPKZ \end{cases}$$

9.25 $$b_3 = \sum_{i+1}^{nx} x_i - PFX_i$$
$$pf_i = \begin{cases} PFC_i\ ; & i = nx \\ const = 1\ ; & i = nx-1, \ldots, 1,\ b_3 \le 0 \\ \dfrac{1}{b_3 \cdot \left(\frac{1}{(KPF_i - PFX_i)} + \frac{1}{b_3}\right)}\ ; & i = nx-1, \ldots, 1,\ b_3 > 0 \end{cases}$$

9.29 $gd = gdt \cdot gdb$

9.30 $$gdb = \left(\frac{\sum x_i \cdot pf_i + d \cdot PF}{z \cdot \left(\frac{KXG}{kz} + \frac{\sum x_i \cdot pf_i + d \cdot PF}{kz} + \frac{KXG}{z} + \frac{\sum x_i \cdot pf_i + d \cdot PF}{z} \right)} \right)$$

9.31 $$kz = \begin{cases} LGL \cdot (\sum x_i \cdot pf_i + d \cdot PF)^{MGL} \ ; & \sum x_i \cdot pf_i + d \cdot PF \leq WPKZ \\ KZMIN + LGH \cdot (\sum x_i \cdot pf_i + d \cdot PF)^{MGH} \ ; & \sum x_i \cdot pf_i + d \cdot PF > WPKZ \end{cases}$$

3.5 Zooplankton

Change =
+ growth of zooplankton
- mortality

12.0 $$\frac{dz}{dt} = zwa - zmo \qquad \left[\frac{g}{m^3} \cdot d \right]$$

12.1 $$zwa = wz \cdot z$$

12.2 $$egg = \begin{cases} \frac{DTMIN}{dt} \ ; & dt \geq DTMIN \\ 1 \ ; & dt < DTMIN \end{cases}$$

12.3 $$dt = \exp\left(DTA - DTB \cdot \ln(temp) - DTC \cdot (\ln(temp))^2 \right)$$

12.4 $$wz = (assiz - rz) \cdot egg$$

12.5 $$assiz = az \cdot \left(\sum g_i \cdot UXZ_i + g_d \cdot UXZD \right) \ ; \quad i = 1, \ldots, nx$$

12.6 $az = \left(AZMAX - \frac{AZMAX - AZMIN}{GMAX} \cdot g \right)$

12.7 $rz = rzg \cdot rzt$

12.8 $rzg = \left(\frac{RZOPT - RZMIN}{GMAX} \cdot g + RZMIN \right) \cdot \frac{1}{RZOPT}$

12.9 $rzt = (RZOPT - RZTMIN) \cdot \left(\frac{temp}{TOPTZ} \right)^2 + RZTMIN$

12.11 $zmo = mortz \cdot z$

12.12 $mortz = (MOMIN + MOT \cdot temp) \cdot \frac{z}{KMO + z}$

3.6 Oxygen

15.1 $T_k = temp + 273.5$

$$sat = \exp(-139.34411 + 157570.1 \cdot T_k - 66423080 \cdot T_k^2 + 12438000000 \cdot T_k^3 - 862194900000 \cdot T_k^4)$$

Change =
+ netto oxygen production by phytoplankton
- oxygen consumption (without respiration part of phytoplankton)

16.0 $\frac{do}{dt} = oprod - okons \qquad \left[\frac{g}{m^3} \cdot d \right]$

16.4 $oprod = \sum \left(\frac{wx_i}{YOX} \cdot x_i \right) \; ; \quad i = 1, \ldots, nx$

16.5 $okons = \frac{lo}{YOX}$

16.6 $lo = \sum xsed_i + rz \cdot z + lsez \; ; \quad i = 1, \ldots, nx$

16.7 $\sum xsed_i = \sum \left(\frac{VS_i \cdot x_i \cdot miner_i}{zmix} \right) + \frac{VD \cdot d \cdot miner_d}{zmix} \; ; \quad i = 1, \ldots, nx$

16.8 $miner_i = \begin{cases} KMINER \cdot \frac{zmix}{VS_i} \; ; & miner_i \leq 1 \\ 1 \; ; & miner_i > 1 \end{cases} \; ; \quad i = 1, \ldots, nx$

$miner_d = \begin{cases} KMINER \cdot \frac{zmix}{VD} \; ; & miner_d \leq 1 \\ 1 \; ; & miner_d > 1 \end{cases} \; ; \quad d = \max\left(1, KMINER \cdot \frac{zmix}{VD}\right)$

16.9 $lsez = seza \cdot YOK \cdot \frac{ased}{v}$

16.10 $seza = SEZMAX \cdot \exp(0.08 \cdot temp) \cdot \frac{o}{KSEZA + o}$

3.7 Detritus

Change =
- grazing by zooplankton

18.0 $\frac{dd}{dt} = -dgraz \qquad \left[\frac{g}{m^3} \cdot d\right]$

18.4 $dgraz = g_d \cdot z$

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